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BCF-003-1015001 Seat No. ________

B. Sc. (Sem. V) (CBCS) (W.E.F. 2016) Examination

August – 2021

Mathematics : Paper - V (A)

(Mathematical Analysis - I and Abstract Algebra - I)

(New Course)

Faculty Code : 003

Subject Code : 1015001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

1) Attempt any FIVE questions out of TEN questions.

2) Numbers written to the right side indicates full marks of the question.

1. (a) **Answer the following questions briefly :** **4**
   1) Define Dense Set.
   2) Define Discrete Metric Space.
   3) Define Limit Point of Metric Space.
   4) Define Interior Point of Metric space.

   (b) Prove that finite intersection of finite Open Set of Metric Space is Open Set. **2**

   (c) Prove that every convergent sequence is Cauchy Sequence. **3**

   (d) In usual notation prove that $\overline{E}$ is closed set. **5**

2. (a) **Answer the following questions briefly :** **4**
   1) Give an example of neither open nor closed set in standard metric space.
   2) If $E = (1,5)$ is a subset of metric space $\mathbb{R}$ then $E' =$ ______ ?
   3) If $(\mathbb{R}, d)$ is a usual metric space, then find $(1,2)^{\circ}$.
   4) If $\mathbb{Q}$ is a subset of metric space $\mathbb{R}$ then $\overline{\mathbb{Q}} =$ _____.

   (b) Prove that $\mathbb{N}' = \emptyset$. **2**

   (c) In usual notation prove that (R, d) is metric space. **3**

   (d) If (X, d) is metric space, then show that $\left(X, \frac{d}{1+d}\right)$ is also metric space. **5**

3. (a) **Answer the following questions briefly :** 4

1) Define Refinement.
2) Define Riemann Integration.
3) State Darboux's Theorem.
4) Define Oscillatory Sum.

(b) Prove that $\int_{\underline{a}}^{b} f(x)dx \leq \int_{a}^{\overline{b}} f(x)dx$ 2

(c) If $f, g \in R_{[a,b]}$ then prove that $f + g \in R_{[a,b]}$. 3

(d) State and Prove Necessary and Sufficient Condition for a bounded function f on $[a, b]$ to be R–Integrable. 5

4. (a) **Answer the following questions briefly :** 4

1) If $P = \{1, 7.5, 15.5, 20\}$ is partition of $[1.20]$ then find $\| P \| =$ _____.
2) Let $f(x) = x, x \in [0,1]$ and $P = \left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$ be a partition of $[0,1]$ then compute $U(P, f)$.
3) $\int_{-1}^{1} \sin(x)\, dx =$ _____
4) What are the supremum and infimum of set $S = \left\{\frac{1}{n} \mid n \in \mathbb{N}\right\}$?

(b) If function f defined as : $f(x) = \begin{cases} 0\, ; x \in \mathbb{Q} \\ 1\, ; x \notin \mathbb{Q} \end{cases}$ Then Show that f is not R–Integral over $[a, b]$. 2

(c) State and Prove Fundamental Theorem of Integration. 3

(d) For $0 < x < \frac{\pi}{2}$, Show that $f(x) = cos(x)$ is R–Integrable and Find $\int_{0}^{\frac{\pi}{2}} \cos(x)\, dx$. 5

5. (a) **Answer the following questions briefly :** 4

1) Define Norm of the Partition.
2) In usual notation define $L(P, f)$.
3) State Second Mean Value Theorem for Integration of Bonnett's Form.
4) State First Mean Value Theorem,

(b) Prove that f is continuous in $[a, b]$ then prove that $f \in R_{[a,b]}$. 2

(c) Evaluate : $\lim_{n\to\infty} n \sum_{r=0}^{n-1} \frac{1}{n^2+r^2} = \frac{\pi}{4}$ 3

(d) State and Prove General form of First Mean Value Theorem. 5

6. (a) **Answer the following questions briefly:** 4

1) Find the order of element 4 of $(\mathbb{Z}_6, +_6)$ and also find total number of generators of $(\mathbb{Z}_6, +_6)$.
2) Define Klein's Group.
3) Define Cyclic group.
4) Define General Linear Group

(b) State and Prove Reversal Law's for a group. 2

(c) Let G be a group and $a, b \in G$ then prove that the equations $a * x = b$ and $y * a = b$ have unique solutions. 3

(d) Prove that every subgroup of cyclic group is cyclic. 5

7\. (a) **Answer the following questions briefly:** **4**

1) Define Subgroup.
2) Define Coset.
3) Define Index of subgroup.
4) Define Proper and Improper subgroup.

(b) Show that a non–empty subset H of Group G is subgroup of G iff $ab^{-1} \in H$. **2**

(c) If $H \leq G$ for $a, b \in G$ then prove that $H_a \neq H_b \Rightarrow H_a \cap H_b = \emptyset$. **3**

(d) State and Prove Lagrange's Theorem. **5**

8\. (a) **Answer the following questions briefly:** **4**

1) Give an example of non–abelian group
2) Find the order of permutations : $f = (1\ 3\ 5\ 2) \in S_5$ *and* $g = (1\ 2\ 3\ 4) \in S_5$.
3) Find $f \cdot g$ where $f = (1\ 3\ 5)$ and $g = (2\ 4) \in S_6$.
4) Give an example of non-cyclic group which is an abelian

(b) Write all the element of S_3. **2**

(c) Prove that any two disjoint cycle in S_n is commutative. **3**

(d) Define Alternating group A_n, Show that $A_n (n \geq 2)$ is subgroup of S_n of order $\frac{n!}{2}$. **5**

9\. (a) **Answer the following questions briefly:** **4**

1) Define normal subgroup.
2) Define quotient group.
3) Define isomorphism of group.
4) Define simple group.

(b) Define Translation, Invariant and Transposition. **2**

(c) Let $H \leq G$ and $K \leq G$ then Prove that $K \cap H$ is normal subgroup of K if H is normal subgroup of G. **3**

(d) State and Prove Cayley's Theorem. **5**

10\. (a) **Answer the following questions briefly:** **4**

1) Define automorphism of group.
2) Define Quaternion group.
3) Define inner automorphism of group.
4) Give an example of smallest simple group.

(b) Prove that intersection of two normal subgroup of group is normal subgroup of group. **2**

(c) Show that the mapping $f : (\mathbb{R}, +) \rightarrow (\mathbb{R}^+, \times)$ is defined by $f(x) = e^x$; $\forall x \in \mathbb{R}$ is an isomorphism **3**

(d) Prove that a subgroup H of group G is normal subgroup iff $(H_a)(H_b) = H_{ab}$; $\forall a, b \in G$. **5**
